On the Optimal Allocation of Security Listings to Specialists

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Abstract

This paper addresses the question of how securities with correlated payoffs should be allocated to dealers in a specialist system. Using a multi-asset model of an imperfectly competitive market, we examine the effect of alternative security allocations on the specialists' market power and on their adverse selection risk. We demonstrate that specialists are always better off when their portfolios contain securities with highly correlated payoffs, and provide conditions under which risk-averse investors prefer such an allocation as well. Intuitively, this is the case when the investors' order flow is sufficiently informative about the value of the traded securities. We also discuss how the allocation of security listings to specialists affects market liquidity.

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1 Introduction

Many organized financial markets rely on dealers to serve as intermediaries between buyers and sellers. One of the most common arrangements is the specialist system, a hybrid market structure that includes an auction component (e.g., a floor auction or a limit order book) together with one or more designated market makers ("specialists").[1] These dealers, who trade for their own account, have some responsibility for maintaining a "fair and orderly market" in their securities, which typically involves the obligation to continuously post quotes with reasonable depth or to ensure that the quoted bid-ask spread does not exceed a certain level.[2] In return for this responsibility, they receive a preferential treatment in terms of privileged access to order flow, discounted trading fees, or direct monetary compensation from the exchange or the listed firm.[3]

In many specialist markets, barriers of entry for potential dealers make the market-making activity more monopolistic than competitive. While specialists face competition from "upstairs traders," OTC dealers, and crossing networks, their privileged access to information on the content of the limit order book and their ability to condition their participation as a dealer on the size of an order allows them to maintain some monopoly power despite the presence of these other traders.[4] Of course, this power is limited by the fact that securities

[1]The classic example of an exchange employing such a market making system is the New York Stock Exchange (NYSE). For decades, each security listed on the NYSE was traded by a single specialist (Seligman, 2003). In 2008, the NYSE moved to a multi-dealer market structure, creating a new class of market maker (called "supplemental liquidity provider") that operates outside the exchange and competes with specialists (who are now called "designated market makers"). In recent years, a number of European exchanges have introduced designated market makers to supply liquidity for at least some stocks traded in their electronic limit order markets (Skjeltorp and Ødegaard, 2012).

[2]For example, designated market makers on the NYSE are obligated to quote at the national best bid or offer a specified percentage of the time, and to facilitate price discovery at the open, close, and in periods of significant imbalances.

[3]The NYSE, for example, pays 30 cents per 100 traded shares to designated market makers. The rebate for supplemental liquidity providers is 15 cents per 100 shares.

[4]For example, Hasbrouck and Sofianos (1993) report significantly positive trading profits for NYSE specialists. Ready (1999) and Harris and Panchapagesan (2005) provide empirical evidence that specialists are able to benefit from their informational advantage over other market participants. Interestingly, Anand and Subrahmanyam (2008) find that even in the completely electronic and highly transparent market of the Toronto Stock Exchange, intermediaries earn significant positive returns on their trades.

are imperfect substitutes for one another. This paper explores how the allocation of securities between dealers affects the specialists' market power and, hence, the liquidity of the traded securities.

While each listed security is usually assigned a single specialist, every specialist is in charge of several securities. This raises the question of how securities with correlated payoffs should be optimally allocated between market makers. For example, a stock exchange could decide to allocate all of its internet stocks to one dealer and all of its energy stocks to another. Alternatively, it could also assign a relatively diverse and uncorrelated set of stocks to each specialist. Each of the two scenarios has its own welfare implications. In the former scenario, each market maker possesses superior information about her own industry, and is therefore probably in a good position to distinguish between informational and non-informational trading. In the latter scenario, the competition among market makers for order flow in highly correlated securities makes prices more attractive to market participants who trade for liquidity reasons. This example shows that there is no simple answer. In a multi-asset economy, the correlation structure of security payoffs affects both the specialist's adverse selection risk as well as her market power.

This paper develops a noisy rational expectations model of a specialist market that enables us to study this tension between market power and information precision. Payoffs of securities within an industry are assumed to be correlated, whereas payoffs of securities in different industries are assumed to be independent. Specialists are risk neutral and have monopoly control over their assigned securities. Investors have informational as well as non-informational motives for trade. Their informational motive stems from the fact that they observe a noisy signal about security payoffs prior to trading. Their non-informational trading activity is driven by the fact that they receive a stochastic stock endowment that they need to hedge. The indirect competition among specialists is modeled as a Cournot game under asymmetric information.

Our analysis shows that specialists are better off when their portfolios contain securities with highly correlated payoffs. Such an allocation offers two advantages to them. First, by making a market for several securities that are considered close substitutes by investors, a specialist can avoid competing with other dealers for order flow in these securities, which allows her to extract higher rents from investors. Second, observing the investors' demand for securities whose fundamentals are driven by the same underlying factor enables the specialist to better distinguish between informational and non-informational trades. This reduces her adverse selection risk.

Interestingly, we find that investors may favor such a concentrated allocation as well. While investors suffer from a lack of competition in this case, they benefit from a reduction in hedging costs associated with better informed specialists. The better informed a specialist is, the less information she can infer from the observed order flow for a given stock, and thus the less sensitive her price will be to the investors' demand for that stock. A smaller informational gap between investors and specialists therefore allows investors to hedge their endowment risk more efficiently: they can fulfill their trading needs without adversely affecting stock prices too much.

The reduction in hedging costs has to be weighed against the specialists' increased market power. Our results indicate that the trade-off between these two opposing forces depends on the "within-industry" correlation of security payoffs (as compared to the "between-industry" correlation which we normalize to zero). For high levels of correlation, the adverse price effect due to the specialists' increased market power dominates the reduction in hedging costs and investors prefer an allocation of highly correlated securities to different specialists. For low levels of correlation, the opposite may be true: if the order flow observed by specialists is sufficiently informative about future asset payoffs, investors are better off when securities in an industry are assigned to the same specialist.

The allocation of security listings to specialists also has implications for market liquidity. Our analysis shows that the trading volume is, on average, higher when securities in the same industry are allocated to the same specialist. The price impact of an order depends again on the "within-industry" correlation of security payoffs: for low levels of correlation, it is typically higher in the case where each specialist is assigned securities from different industries, whereas for high levels of correlation, it is higher in the case where each specialist is assigned securities from the same industry.

Our results also contribute to the debate on how to best compensate liquidity providers for their services. In electronic limit order markets, designated market makers typically receive monetary compensation for taking on certain affirmative obligations from the exchange or the listed company (Charitou and Panayides, 2008). Our analysis suggests that granting them privileged access to information on the content of the limit order book may be an attractive alternative: this informational advantage relative to other market participants not only allows designated market makers to maintain some monopoly power, but may also improve market liquidity and enhance social welfare.

Starting with Glosten (1989), a number of papers have studied the behavior of imperfectly competitive market makers under adverse selection.[5] Most of them, however, focus on the single-asset case and, thus, do not shed any light on the question of how securities should be allocated to dealers. A notable exception is Hagerty (1991) who models the specialist system as a monopolistically competitive market. She shows that increasing the number of securities generally lowers bid-ask spreads, because increased hedging opportunities for investors reduce any particular specialist's monopoly power.

Gehrig and Jackson (1998) extend Hagerty's model by allowing for a more general correlation structure of security payoffs. They show that diversified specialist portfolios are socially optimal when securities trade as substitutes, whereas concentrated portfolios are optimal when they trade as complements.

Our approach differs from these papers in that specialists face an adverse selection problem

[5]Glosten (1989) analyzes the case of a monopolist specialist and shows that such a setting may produce more efficient outcomes than a perfectly competitive market-making sector. Biais, Martimort, and Rochet (2000) extend Glosten's model by incorporating multiple liquidity suppliers who offer competing trading mechanisms to a risk-averse agent with private information. They show that, for any finite number of competitors, prices are above marginal costs and liquidity suppliers earn positive expected profits.

when they set their prices. In contrast, Hagerty (1991) and Gehrig and Jackson (1998) assume that investors and specialists share the same information about the value of the traded assets. Consequently, investors cannot benefit from the increased information flow associated with concentrated specialist portfolios and therefore always prefer independent specialists when securities trade as substitutes.

Our paper is also related to the literature that studies the optimal listing choice of firms. While most of this literature focuses on differences in the structure or rules of the exchanges, Baruch and Saar (2009) argue that differences in the return patterns of securities traded on these exchanges may be an important determinant of the listing decision. In a multi-asset version of the Kyle (1985) model with two segmented markets and a competitive market-making sector, they show that a stock is more liquid when it is listed on the market where "similar" securities are traded.[6] Our paper complements their analysis by showing that, in a specialist system, the correlation of security payoffs not only affects the information that dealers can extract from the observed order flows, but also their market power.

Corwin (2004) studies the allocation process of new listings on the NYSE. Using a discrete-choice logit model, he finds that, besides the past performance of specialists, their portfolio concentration in the industry of the new listing is an important factor in the allocation decision. The effect is particularly pronounced for allocations prior to 1997 when the decision was made solely by the exchange's Allocation Committee.[7] This is consistent with our results. Since allocations prior to 1997 are likely to reflect primarily the interests of the specialist and the exchange, our model suggests that during this period specialist portfolios should be concentrated with respect to industry and security type.[8]

[6] In a similar setting, Baruch, Karolyi, and Lemmon (2007) demonstrate that the trading volume of a cross-listed stock is higher on the exchange in which its return is more highly correlated with returns of other assets traded on that market. See also Caballé and Krishnan (1994) who provide a characterization of linear equilibria in a Kyle (1985)-type model with multiple assets and general covariance matrices of asset payoffs, noise trading, and private signals.

[7] In March of 1997, the NYSE revised its Allocation Policy to give listing firms the option to choose their specialist from a subset of three to five specialist units selected by the Allocation Committee (New York Stock Exchange, Inc., 1998, 2001).

[8] After the policy change in 1997 when listing firms were given greater influence over the choice of their

Examining stock reassignments on the NYSE, Anand, Chakravarty, and Chuwonganant (2009) report that industry concentrations of specialist portfolios also play a prominent role in the decision of specialist firms to reassign stocks from one individual specialist portfolio to another. Portfolios affected by reassignments have significantly lower industry concentrations before the reassignments than portfolios that were unaffected by the reassignments. After the reassignment, affected and unaffected portfolios have very similar levels of industry concentration. This is consistent with our finding that specialists prefer assets with correlated payoffs. Anand, Chakravarty, and Chuwonganant also document an improvement in liquidity following the reassignment: reassigned stocks experience a decline in their quoted and effective spreads around the reassignment date. Our model predicts such an increase in market liquidity for stocks with moderate levels of within-industry correlation.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 characterizes the optimal strategies of the different agents and derives equilibrium prices under two different market-making scenarios. Section 4 compares the investors' and the specialists' welfare under these two scenarios and discusses implications for market liquidity. Section 5 summarizes our contribution and concludes. All proofs are contained in the Appendix.

2 The Model

The model analyzes a two-date exchange economy. Agents trade at date 0 and consume at date 1. There are two types of agents in the economy: risk-averse investors who possess private information about the state of the economy and their own endowment, and risk-neutral market makers. The structure of the economy is common knowledge.

specialist, our model predicts that liquidity concerns will lead to more diversified specialist portfolios (especially when stock returns within industries are highly correlated).

2.1 Investment opportunities

There are five securities available for trading at date 0, which pay off in the economy's single consumption good. The first security is a risk-free bond that yields a sure payoff of 1 at date 1. The bond is in perfectly elastic supply and its interest rate is normalized to zero. The remaining four securities are risky stocks. Shares of the stocks are infinitely divisible and are traded competitively in the stock market. Each share of stock n pays a liquidation value of v_n at date 1, which is unknown at date 0. Let v denote the vector of random payoffs at date 1, and P the vector of equilibrium share prices at date 0.

For the sake of tractability, we make the following simplifying assumptions about security payoffs. Each firm issuing shares belongs to one of two industries, a or b, whose lines of business are unrelated. Specifically, we assume that the payoff vector of the first two stocks, $v_a = (v_{a1}, v_{a2})$, is stochastically independent of that of the remaining two stocks, $v_b = (v_{b1}, v_{b2})$. On the other hand, profits of firms operating in the same industry are correlated: $\text{Corr}\,[v_{k1}, v_{k2}] = \rho_k \in (-1, 1),\ k \in \{a, b\}$. Positively correlated securities are substitutes, while negatively correlated securities are complements. For simplicity, we assume that the payoff correlation is identical across industries, i.e., $\rho_a = \rho_b = \rho$.

2.2 Investors and market makers

Our economy is populated by two types of market participants. The first type, which we call investors, receives (a vector of) signals s about the liquidation values v prior to trading:

$$s = v + \epsilon, \tag{1}$$

where the error term ϵ is independent of the payoff vector v. This gives rise to their informational trading. In addition, investors experience an endowment shock, which creates a non-informational motive for trade. For simplicity, we assume that all investors have identical endowment shocks, denoted by the vector $\bar{x}$.[9] The quantity $\bar{x}_n$ can be positive in which case investors hold a long position in stock n, or negative in which case they hold a short position.

All investors are assumed to have mean-variance preferences with an identical risk aversion coefficient of γ. They maximize their expected utility from date-1 consumption given by:

$$U_I = \mathbb{E}\left[W_I | \mathcal{F}_I\right] - \frac{\gamma}{2} \operatorname{Var}\left[W_I | \mathcal{F}_I\right], \tag{2}$$

where $\mathcal{F}_I$ denotes the investors' information set at date 0 and W_I denotes their wealth at date 1. Note that as long as W_I is normally distributed conditional on $\mathcal{F}_I$, this mean-variance preference specification leads to the same portfolio choice as a negative exponential utility function. In particular, these preferences exhibit constant absolute risk aversion (CARA), implying that the investors' demand for risky assets is unaffected by changes in their initial wealth. The reason why we choose this mean-variance specification is that it simplifies comparisons of the investors' ex ante expected utility across different market-making scenarios.[10]

Investors are assumed to behave competitively. They take equilibrium prices as given even though their aggregate trades affect market prices. Such behavior can be justified by assuming that investors are individually infinitesimal, so that no single trader can influence the price. More precisely, we assume that there is a continuum of identical investors with measure one.[11] As is customary in the rational expectations literature, we assume that investors can submit their orders as a function of the price vector P. Their aggregate demand schedule is denoted by $x(P)$.[12]

[9] A more general distribution of stock endowments can easily be incorporated into the model; however, as long as the aggregate endowment shock is nonzero with positive probability, our basic conclusions remain unchanged.

[10] Biais, Martimort, and Rochet (2000) use the same preference specification to analyze the efficiency of their trading mechanism at the ex ante stage.

[11] Normalizing the measure to one is without loss of generality in our model, since having one investor with risk aversion parameter γ is equivalent to having N investors with risk aversion parameter γ/N.

[12] In a multi-asset economy, this assumption implies that the demand schedule cannot be implemented with standard limit orders, since the demand for a security is contingent on the price of other securities (e.g., Admati, 1985; Bernhardt and Taub, 2008).

The second type of agents are risk-neutral market makers.[13] For simplicity, we only consider the case of two competing market makers. Each market maker $m \in \{1, 2\}$ is assigned a set of stocks H_m, for which she exclusively controls all transactions and acts as a counterpart to all trades. No trades are permitted to occur directly between investors.[14]

Market makers do not observe the signal s or the endowment shock $\bar{x}$. Their only source of information is the investors' aggregate demand $x(P)$. A key assumption of our model is that market makers observe only the aggregate order flow in their own stocks before setting prices (i.e., market maker m only observes $x_n(P)$ if $n \in H_m$).[15] There are several reasons for why specialists may be better informed about the order flow of securities for which they make a market (as compared to securities for which they do not make a market). For example, they may have privileged access to the limit order book of their assigned securities. Alternatively, their repeated interactions with floor brokers may enable them to better differentiate between informed and uninformed orders.[16]

The indirect competition among market makers—resulting from a nonzero correlation of security payoffs—is modeled as a Cournot game under asymmetric information. After observing the aggregate demand schedule, market makers simultaneously choose the quantities that they are willing to trade. To distinguish these quantities from the investors' demand schedule, we denote market maker m's strategy by $\hat{x}_{H_m}$. The assumption that market makers compete in quantities rather than in prices can be justified by the fact that their capacity to trade is limited by their inventory position. As Kreps and Scheinkman (1983) noted, introducing capacity constraints into a game of imperfect competition in which agents first

[13]While the assumption of risk neutrality simplifies the comparison of the different market-making scenarios, it is not crucial to our main results. Introducing risk-averse market makers changes the optimal security allocation in favor of better diversified specialist portfolios, but does not alter our basic conclusions.

[14]This assumption is without loss of generality since investors' trades are perfectly correlated in our model.

[15]Chowdhry and Nanda (1991), Baruch, Karolyi, and Lemmon (2007), and Baruch and Saar (2009) make the same assumption with respect to market makers operating on different exchanges.

[16]Benveniste, Marcus, and Wilhelm (1992) present a model of a specialist market in which the specialist's ability to differentiate between informed and uninformed orders can improve the welfare of exchange members as well as the terms of trade for the investing public.

choose capacities and then prices yields, under some conditions, Cournot outcomes.[17] We want to point out, however, that quantity competition is not critical to our analysis. It is straightforward to show that having market makers compete in prices yields qualitatively similar results.[18]

2.3 Distributional assumptions

For tractability, we assume that the random vectors v, ϵ, and $\bar{x}$ are independently normally distributed with zero means. This implies that the investors' signals s are unbiased forecasts of v. We further assume that the forecast errors ϵ are i.i.d. with variance σ_ϵ^2: $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2 I_4)$, where I_4 denotes the 4×4 identity matrix. Similar assumptions are made about the distribution of the endowment shocks: $\bar{x} \sim \mathcal{N}(0, \sigma_x^2 I_4)$. A more general correlation structure of the stock endowments can easily be incorporated into the model, but leads qualitatively to the same results.

As specified above, the liquidation values v_a and v_b are stochastically independent. In order to keep the model simple, we assume that v_a and v_b have the same covariance matrix given by:[19]

$$\Sigma_v = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}. \tag{3}$$

2.4 Timing of events

The timing of events in our model is as follows. First, investors observe their private signals s and endowment shocks $\bar{x}$. Based on their private information, investors then submit their

[17] Kremer and Polkovnichenko (2000) present a single-asset model of a financial dealership market in which the position that a dealer can take is limited by her capital. By endogenizing the dealer's capital choice, Kremer and Polkovnichenko demonstrate that these capacity constraints lead to a Cournot equilibrium in which dealers maintain some market power and earn positive profits.

[18] The reason why the equilibrium outcome of the Cournot game is different from that of the Bertrand game is that, if $\rho \neq 0$, the elasticity of the inverse demand is not equal to the inverse of the demand elasticity.

[19] Note that normalizing the variance of v_n to unity is without loss of generality, since only the ratios between the payoff variance and the variances σ_ϵ^2 and σ_x^2 enter our calculations.

demand schedules to the market makers. These market makers, in turn, simultaneously choose quantities based on whatever information they can extract from the observed aggregate demand. Finally, trading takes place, liquidation values are revealed, and profits are realized.

3 Equilibrium Trades and Prices

In this section, we solve for the equilibrium of the economy defined above. The equilibrium concept we use is that of a Perfect Bayesian Equilibrium (PBE). Formally, an equilibrium is defined by a set of demand functions $\{x^i(P)\}$, $i \in [0,1]$, and trading strategies $\{\hat{x}_{H_m}\}$, $m = 1, 2$, such that:

(i) the trades specified by investor i's demand function $x^i(P)$ maximize her expected utility of consumption based on her information set $\mathcal{F}_I$;

(ii) the trading strategy $\hat{x}_{H_m}$ maximizes market maker m's expected utility, conditional on her information set $\mathcal{F}_m$, the investors' aggregate demand function $x(P)$, and the other market maker's trading strategy $\hat{x}_{H_l}$, $l \neq m$.

Equilibrium prices are given by $P = x^{-1}(\hat{x}_{H_1}, \hat{x}_{H_2})$, where x^{-1} denotes the inverse of the investors' aggregate demand function:[20]

$$x(P) = \int_0^1 x^i(P)di. \tag{4}$$

The first condition of our definition simply states that each investor chooses his demand for risky securities optimally. The second condition ensures that the quantities chosen by each market maker are a best response to the investors' demand and the other market maker's strategy.

[20] As will become clear in Section 3.1, the Jacobian of the aggregate demand function x is negative definite. Thus, the inverse of x exists.

The problem is considerably simplified by noting (i) that investors take equilibrium prices as given, and (ii) that investors have superior information about asset payoffs relative to market makers (in the sense that the information set of the latter is a subset of the information set of the former). These assumptions immediately imply that, when choosing her optimal strategy $x^i(P)$, investor i does not take into account the other players' strategies. Moreover, since all investors have the same preferences, information sets, and endowments, their trading strategies are identical. Thus, each investor's optimal demand function $x^i(P)$ coincides with the aggregate demand function $x(P)$. The problem of the two market makers is therefore reduced to that of simultaneously picking quantities on the aggregate inverse demand schedule x^{-1}.

3.1 The investors' problem

Each investor chooses her portfolio of assets to maximize her expected utility subject to her budget constraint and available information. At date 0, an investor's information set contains the signal vector s, her stock endowment $\bar{x}$, as well as the equilibrium price vector P.[21] Let $\mu_I = \mathbb{E}\left[v|\mathcal{F}_I\right]$ denote the vector of expected payoffs conditional on the information set $\mathcal{F}_I = \{s, \bar{x}, P\}$, and $\Sigma_I = \mathrm{Var}\left[v|\mathcal{F}_I\right]$ denote the conditional covariance matrix of asset payoffs. Then, each investor faces the following optimization problem:[22]

$$\max_{x} \; x^{\mathrm{T}}(\mu_I - P) + \bar{x}^{\mathrm{T}}\mu_I - \frac{\gamma}{2}\,(x+\bar{x})^{\mathrm{T}}\Sigma_I(x+\bar{x}) \tag{5}$$

Maximizing this quadratic objective function yields:

$$x(P) = \frac{1}{\gamma}\,\Sigma_I^{-1}(\mu_I - P) - \bar{x}. \tag{6}$$

[21]The information set contains the price vector P since investors can submit their demands as a function of the price.

[22]Superscript T denotes the transpose operation.

Note that $x(P)$ is a global maximum, since the second derivative of (5) with respect to x is equal to the matrix $-\gamma\Sigma_I$, which is negative definite.

In order to calculate the conditional moments of the payoff vector v, note that v is independent of the endowment shock $\bar{x}$. Further, since market makers do not have any private information about v, equilibrium prices cannot reveal any additional information to investors. Thus, conditioning on $\mathcal{F}_I$ is equivalent to conditioning just on the signal vector s. Using the fact that v and s are jointly normally distributed, we can therefore calculate μ_I and Σ_I from the multivariate projection theorem:[23]

$$\mu_I = \begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix}, \tag{7}$$

$$\Sigma_I = \begin{pmatrix} \Sigma_{v|s} & 0 \\ 0 & \Sigma_{v|s} \end{pmatrix}, \tag{8}$$

where:

$$\mu_k = \Sigma_v \left(\Sigma_v + \sigma_\epsilon I_2\right)^{-1} s_k, \qquad k \in \{a, b\}, \tag{9}$$

$$\Sigma_{v|s} = \Sigma_v - \Sigma_v \left(\Sigma_v + \sigma_\epsilon I_2\right)^{-1} \Sigma_v, \tag{10}$$

and I_2 denotes the 2×2 identity matrix. The above equations show the well-known result that for jointly normally distributed random variables, the conditional expectation is a linear function of the conditioning variable, whereas the conditional variance does not depend on the conditioning variable.

The expression for the conditional covariance matrix Σ_I in equation (8) implies that the investors' demand functions for stocks in industries a and b are independent. This is not surprising, given that the security payoffs v_a and v_b and the signals s_a and s_b are independent.

[23] See, e.g., Anderson (1984), chapter 2.

Substituting the expressions for the conditional moments of v into equation (6) and letting θ denote the combined demand effect of the signal s and the endowment shock $\bar{x}$, i.e.:

$$\theta_k = \mu_k - \gamma\,\Sigma_{v|s}\,\bar{x}_k, \qquad k \in \{a, b\}, \tag{11}$$

we can rewrite the investors' demand function more conveniently as follows:

$$x_k(P_k) = \frac{1}{\gamma}\,\Sigma_{v|s}^{-1}\,(\theta_k - P_k), \qquad k \in \{a, b\}. \tag{12}$$

The right-hand side of this equation shows the familiar result that CARA preferences under normal distributions of payoffs and signals lead to linear optimal demand functions.

3.2 The market makers' problem

In order to answer the question of how to optimally allocate security listings to specialists, we analyze the welfare effects of two different market-making scenarios. In the first scenario, each market maker is in charge of two stocks in the same industry. Since the investors' demand functions for the two industries, x_a and x_b, are independent, there is no competition for order flow in this situation. Hence, each market maker acts as a monopolist. We will therefore refer to this case as the monopolistic market-making scenario.

In the second scenario, which we will call the duopolistic market-making scenario, each market maker is assigned one stock from each of the two industries. In this case, market makers compete for order flow, making prices more attractive to investors. The intensity of competition depends on the correlation coefficient ρ.

We will demonstrate that, for both market-making scenarios, there exists a unique equilibrium in which prices are linear functions of θ, the market makers' noisy observation of the the investors' signal s.

3.2.1 Monopolistic market making

In this section, we consider the scenario in which each market maker sets prices for both securities of one of the two industries. Note that in this case, it makes no difference whether market makers choose prices or quantities, since there is no strategic interaction between them. We will therefore state the specialist's problem in terms of prices. Since both industries are ex ante identical, we will focus on one of them and from now on drop the industry subscript (i.e., $x(P)$ now stands for the demand schedule $(x_1(P), x_2(P))^{\mathrm{T}}$ and the price vector is defined as $P = (P_1, P_2)^{\mathrm{T}}$).

The market maker's trading profit is equal to $x(P)^{\mathrm{T}}(P - v)$, where the investors' demand function $x(P)$ is given by equation (12). When choosing prices that maximize her expected profit, the market maker can condition on the observed demand schedule $x(P)$. Thus, her profit is normally distributed and the profit-maximizing prices are found by solving the following optimization problem:

$$\max_{P} \quad \frac{1}{\gamma} \left(\theta - P\right)^{\mathrm{T}} \Sigma_{v|s}^{-1} \left(P - \mathbb{E}\left[v|\mathcal{F}_m\right]\right) \tag{13}$$

Note that observing $x(P)$ is informationally equivalent to observing θ (as defined in equation (11)), as long as the matrix $\Sigma_{v|s}$ has full rank (i.e., as long as $|\rho| < 1$). Moreover, since v and θ are jointly normally distributed, the expected value of v conditional on θ follows immediately from the projection theorem:

$$\mathbb{E}\left[v|\theta\right] = \mathbb{E}\left[\mu|\theta\right] = \Sigma_\mu \, \Sigma_\theta^{-1} \, \theta, \tag{14}$$

where:

$$\Sigma_\mu \;=\; \mathrm{Var}\left[\mu\right] \;=\; \Sigma_v - \Sigma_{v|s}, \tag{15}$$

$$\Sigma_\theta \;=\; \mathrm{Var}\left[\theta\right] \;=\; \Sigma_\mu + \gamma^2 \, \sigma_x^2 \, \Sigma_{v|s}^2. \tag{16}$$

From the first order condition, the optimal price function is found to be linear in θ and, hence, in μ and $\bar{x}$:

$$P^M = \delta^M \, \theta, \tag{17}$$

where:[24]

$$\delta^M = \left(I_2 + \Sigma_\mu \, \Sigma_\theta^{-1}\right) / 2. \tag{18}$$

It is easily verified that P^M is the unique maximum, since the second derivative of the market maker's objective function in (13) with respect to P is equal to the matrix $-\frac{2}{\gamma}\, \Sigma_{v|s}^{-1}$, which is negative definite. This, in fact, establishes the following result.

Proposition 1. *Suppose that each market maker is assigned two stocks from the same industry (i.e., $H_1 = \{a1, a2\}$ and $H_2 = \{b1, b2\}$). Then, the unique equilibrium is characterized by demand functions as specified in (12) and by prices as specified in (17).*

3.2.2 Duopolistic market making

When each market maker is in charge of one stock of each of the two industries, market makers compete for order flow, since securities are now (imperfect) substitutes for each other (assuming that $\rho > 0$). Thus, a low price of stock 2 reduces the demand for stock 1, and vice versa. When making their portfolio decisions, market makers therefore have to take into account that prices are affected by their rival's strategy. Again, since x_a and x_b are independent, we will only consider one industry (and drop industry subscripts).

In this market-making scenario, market maker 1's optimal trading strategy in security 1, $\hat{x}_1$, solves the following problem:

$$\max_{\hat{x}_1} \;\; \hat{x}_1 \, \mathbb{E}\left[x_1^{-1}(\hat{x}) - v_1 | \mathcal{F}_1\right], \tag{19}$$

[24]Note that the matrix δ^M is symmetric. This is not surprising, since the two securities are identical with respect to their payoff, signal, and endowment distributions.

where x_1^{-1} denotes the investors' aggregate inverse demand function and $\hat{x} = (\hat{x}_1, \hat{x}_2)^{\mathrm{T}}$. It is important to note that market maker 1's information set, $\mathcal{F}_1$, is now different from that in the monopolistic case. Since she only observes the demand schedule for one of the securities in an industry, she cannot infer $\theta = (\theta_1, \theta_2)^{\mathrm{T}}$ from x_1, but only a weighted sum of these two random variables. To see this, it is useful to rewrite the investors' demand for security 1, given by equation (12), as follows:

$$x_1(P) = \kappa\big(\theta_1 - P_1 - \rho_{v|s}(\theta_2 - P_2)\big), \tag{20}$$

where $\rho_{v|s} = \mathrm{Corr}\,[v_1, v_2|s] = \Sigma_{v|s,12}/\Sigma_{v|s,11} = \rho\,\sigma_\epsilon^2/(1-\rho^2+\sigma_\epsilon^2)$ and $\kappa = (1-\rho^2+\sigma_\epsilon^2)/(\gamma(1-\rho^2)\sigma_\epsilon^2)$ is a positive constant.[25] Thus, observing x_1 is informationally equivalent to observing the signal:

$$\tau_1 = \theta_1 - \rho_{v|s}\,\theta_2. \tag{21}$$

Clearly, if $\rho \neq 0$, τ_1 is less informative about v_1 than $\theta = (\theta_1, \theta_2)^{\mathrm{T}}$, because θ_1 and θ_2 are not perfectly correlated.

Since τ_1 is the sum of two normally distributed random variables, it is also normally distributed. Moreover, it is easily verified that the joint distribution of v_1 and τ_1 is normal as well. Thus, the conditional expectation of the payoff v_1, $\mathbb{E}\,[v_1|\tau_1]$, can again be calculated from the projection theorem:

$$\mathbb{E}\,[v_1|\tau_1] = \mathbb{E}\,[\mu_1|\tau_1] = \omega_1^{\mathrm{T}}\,\Sigma_\mu\,e_1\,(\omega_1^{\mathrm{T}}\,\Sigma_\theta\,\omega_1)^{-1}\;\tau_1, \tag{22}$$

where $e_1 = (1,0)^{\mathrm{T}}$, $\omega_1 = (1, -\rho_{v|s})^{\mathrm{T}}$, and the covariance matrices Σ_μ and Σ_θ are defined in equations (15) and (16), respectively.

From the investors' aggregate demand function derived in equation (12), it immediately

[25] We use the notation Σ_{ij} to denote the (i,j)th entry of the matrix Σ.

follows that the inverse demand function can be written as:

$$x^{-1}(\hat{x}) = \theta - \gamma\, \Sigma_{v|s}\, \hat{x}. \tag{23}$$

Thus, the investors' inverse demand function for security 1 is given by:

$$x_1^{-1}(\hat{x}) = \theta_1 - \gamma\, \sigma_{v|s}^2 \left(\hat{x}_1 + \rho_{v|s}\, \hat{x}_2\right), \tag{24}$$

where $\sigma_{v|s}^2$ denotes the conditional variance of v_1, $\mathrm{Var}\,[v_1|s] = \Sigma_{v|s,11}$, and $\rho_{v|s}$ denotes the conditional correlation coefficient defined above. Since θ_1 and τ_1 have a joint normal distribution, the conditional expectation of x_1^{-1} is a linear function of the market maker's signal τ_1, her traded quantity $\hat{x}_1$, and the trading strategy $\hat{x}_2$ that she expects her rival to choose:

$$\mathbb{E}\left[x_1^{-1}(\hat{x})|\mathcal{F}_1\right] = \omega_1^{\mathrm{T}}\, \Sigma_\theta\, e_1 \left(\omega_1^{\mathrm{T}}\, \Sigma_\theta\, \omega_1\right)^{-1}\, \tau_1 - \gamma\, \sigma_{v|s}^2 \left(\hat{x}_1 + \rho_{v|s}\, \mathbb{E}\left[\hat{x}_2|\tau_1\right]\right). \tag{25}$$

Substituting this expression and the conditional expectation of v_1 in (22) into the objective function in (19), we derive the first order condition for a maximum of market maker 1's expected profit with respect to $\hat{x}_1$ as:

$$\hat{x}_1 = \frac{1}{2\,\gamma\, \sigma_{v|s}^2}\, \frac{\omega_1^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\, e_1}{\omega_1^{\mathrm{T}}\, \Sigma_\theta\, \omega_1}\, \tau_1 - \frac{\rho_{v|s}}{2}\, \mathbb{E}\left[\hat{x}_2|\tau_1\right]. \tag{26}$$

The second order condition for a maximum is also satisfied: the second derivative of (19) with respect to $\hat{x}_1$, which is equal to $-2\,\gamma\, \sigma_{v|s}^2$, is negative. Indeed, since the second derivative is negative for all possible values of $\hat{x}_1$, (26) is the unique maximum of (19).

Analogous calculations show that market maker 2's optimal trading strategy is given by:

$$\hat{x}_2 = \frac{1}{2\,\gamma\, \sigma_{v|s}^2}\, \frac{\omega_2^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\, e_2}{\omega_2^{\mathrm{T}}\, \Sigma_\theta\, \omega_2}\, \tau_2 - \frac{\rho_{v|s}}{2}\, \mathbb{E}\left[\hat{x}_1|\tau_2\right], \tag{27}$$

where $\tau_2 = \theta_2 - \rho_{v|s}\,\theta_1$, $e_2 = (0,1)^{\mathrm{T}}$, and $\omega_2 = (-\rho_{v|s}, 1)^{\mathrm{T}}$.[26]

The trading strategies in (26) and (27) constitute each market maker's optimal response to the expected strategy of her opponent. In order to solve for a PBE, we have to find the intersection of the expected optimal response functions. The following proposition shows that there exists a unique equilibrium in which market maker m's strategy is a linear function of her signal τ_m.

Proposition 2. *Suppose that each market maker is assigned one stock from each industry (i.e., $H_1 = \{a1, b1\}$ and $H_2 = \{a2, b2\}$). Then, there exists a unique linear equilibrium. The investors' demand functions are as specified in (12). The market makers' trading strategies are given by:*

$$\hat{x}_m = \frac{\omega_m^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\,e_m}{\gamma\,\sigma_{v|s}^2\left(2\,\omega_m^{\mathrm{T}}\,\Sigma_\theta\,\omega_m + \rho_{v|s}\,\omega_1^{\mathrm{T}}\,\Sigma_\theta\,\omega_2\right)}\,\tau_m, \qquad m \in \{1,2\}. \tag{28}$$

Equilibrium prices are linear in θ:

$$P^D = \delta^D\,\theta, \tag{29}$$

where:

$$\delta^D = \begin{pmatrix} 1 - \frac{(1-\rho_{v|s}^2)\,\omega_1^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\,e_1}{2\,\omega_1^{\mathrm{T}}\,\Sigma_\theta\,\omega_1 + \rho_{v|s}\,\omega_1^{\mathrm{T}}\,\Sigma_\theta\,\omega_2} & 0 \\ 0 & 1 - \frac{(1-\rho_{v|s}^2)\,\omega_2^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\,e_2}{2\,\omega_2^{\mathrm{T}}\,\Sigma_\theta\,\omega_2 + \rho_{v|s}\,\omega_1^{\mathrm{T}}\,\Sigma_\theta\,\omega_2} \end{pmatrix}. \tag{30}$$

4 Optimal Allocation of Security Listings

Having solved for the equilibrium price functions under both market-making scenarios, we are now in a position to tackle the central question of this paper, namely of how the allocation of securities to specialists affects the welfare of the different parties.

[26] Since the matrices Σ_μ and Σ_θ are symmetric, note that $\omega_1^{\mathrm{T}}\,\Sigma_\theta\,\omega_1 = \omega_2^{\mathrm{T}}\,\Sigma_\theta\,\omega_2$ and $\omega_1^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\,e_1 = \omega_2^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\,e_2$.

4.1 Market makers' preferred allocation

From the specialists' perspective, the monopolistic scenario offers two advantages. First, since the investors' demand functions for securities in different industries are independent, there is no competition for order flow and specialists can charge the monopoly price. Second, by observing the demand for two securities with correlated payoffs, specialists are better able to distinguish between informational and non-informational trades, which attenuates their adverse selection problem. It is therefore not surprising that specialists strictly prefer the monopolistic market-making scenario for any $\rho \neq 0$, as the following proposition shows.

Proposition 3. *The specialists' ex ante expected profit in the monopolistic market-making scenario strictly exceeds that in the duopolistic scenario for any nonzero correlation of security payoffs. If security payoffs are uncorrelated, specialists are indifferent between the two scenarios.*

4.2 Investors' preferred allocation

Intuitively, one might expect that, for the same reasons, investors favor the duopolistic market-making scenario. This is not always the case, however. While investors benefit from more competitive prices in the duopolistic case, they suffer from the market makers' lack of information, which can make it more costly for them to hedge their endowment risk. In order to see this, note that the investors' ex ante expected utility (before signals and endowments are observed) can be written as:

$$\mathbb{E}U_I = -\mathbb{E}\left[\Pi\right] - \frac{1}{2\gamma}\,\mathbb{E}\left[(\mu_I - P)^{\mathrm{T}}\Sigma_I^{-1}(\mu_I - P)\right], \tag{31}$$

where Π denotes the specialists' aggregate profit. From Proposition 3, we know that the specialists' expected profit—and, hence, the investors' expected loss—is larger in the monopolistic market-making scenario. Thus, for investors to prefer monopolistic market makers, the reduction in hedging costs associated with better informed specialists, which allows investors to hedge their endowment risk more efficiently and thus lowers their optimal risk exposure, must outweigh the increase in the expected trading loss.

To obtain an intuitive understanding of why investors' hedging costs are decreasing in the precision of the market makers' information, ignore for the moment the specialists' market power and consider the perfectly competitive case in which the market price is equal to the stock's expected payoff, i.e., $P = \mathbb{E}[v|\mathcal{F}_m]$. If investors have a positive endowment in a stock, they want to sell some of their shares to reduce their risk. The number of shares that they choose to sell depends on the market price. If the market maker has access to the same information as investors, then the observed negative order flow does not reveal any additional information to her and the price will be equal to $\mu_I = \mathbb{E}[v|s]$, independent of the investors' demand. At this price, investors will find it optimal to liquidate their entire holdings of the stock and, hence, will bear no risk (see the investors' demand function in equation (6)).[27] Thus, investors can hedge their endowment risk at no cost in this case.[28]

If, on the other hand, the market maker cannot directly observe the investors signal s, her beliefs about the stock's liquidation value will be affected by the observed order flow. Not knowing whether the negative demand is caused by a below-average signal or by an above-average endowment shock, the market maker will revise her expectations about the payoff downwards in this case and set a price below the stock's expected value of μ_I.[29] Thus,

[27] This result is driven by our mean-variance preference specification, which ignores the increase in price risk that investors face when market makers have more information about asset payoffs (cf. Hirshleifer (1971), who was the first to point out that changes in equilibrium prices due to public information introduce some additional uncertainty on agents' wealth, which adversely affects their welfare). We want to emphasize, however, that replacing the mean-variance specification by the negative exponential (CARA) utility function $U(W) = -e^{-\gamma W}$ commonly used in REE models does not affect our results qualitatively (although some of our results can only be derived numerically in this case).

[28] This argument shows that even under perfect competition, investors with hedging needs benefit from a reduction in their informational asymmetry vis-à-vis market makers. The intuition for this result is similar to that for why the expected trading loss of noise traders in Kyle (1985)-type models (who are forced to trade a certain quantity for exogenous reasons) is larger, the greater the informational advantage of insiders (and, hence, the greater the informational disadvantage of market makers) is.

[29] The opposite is true if investors have a negative endowment. In this case, a market maker who does not observe s will partially attribute the higher-than-expected demand to an above-average signal and set the price above the stock's expected payoff of μ_I.

the market maker's lack of information causes the price to be negatively correlated with the investors' endowment, making it more costly for investors to unwind their initial position. In equilibrium, investors therefore sell only part of their endowment, trading off the risk from having more shares in their portfolio against a larger trading loss.

The above argument seems to suggest that the investors' hedging costs are higher in the duopolistic case, compared to the monopolistic case, because of the increased informational asymmetry between investors and market makers. However, it ignores the effect that the specialist's market power has on equilibrium prices. In an imperfectly competitive market, specialists can exploit the investors' hedging needs by setting a price above (below) the stock's expected value when investors need to buy (sell) shares. These "mark-ups" are higher in the monopolistic case, since the indirect competition for order flow in the duopolistic case limits the premium that market makers can charge.

The trade-off between market power and information precision depends on the correlation of the securities' liquidation values. If the payoffs are uncorrelated, there is no informational advantage for the monopolist, but also no disadvantage in terms of competition for the duopolist. Thus, the investors' expected utility is the same under both market-making scenarios. As the correlation increases and the assets within an industry become better substitutes for each other, the duopolist faces more competition. At the same time, the quality of her information deteriorates compared to that of the monopolist: the more highly correlated the asset payoffs are, the more useful the observed order flow for asset 2 is in terms of forecasting the payoff of asset 1. This does, however, not mean that the monopolist's informational advantage is monotonically increasing in ρ. In fact, as the correlation coefficient approaches one, the two assets become perfect substitutes for each other and the investors' demand functions for these assets will be linearly dependent.[30] Thus, in the limiting case, the monopolist and the duopolist will have the same information about asset payoffs. It

[30]The same is true when the correlation coefficient converges to minus one. In this case, the assets are perfect complements and the investors' demand functions for these assets are identical.

is therefore not surprising that, for high values of ρ, the competition effect dominates the information effect, making the duopolistic scenario more appealing to investors.

For low values of the correlation coefficient ρ, investors may end up preferring either scenario. If the monopolist's informational advantage is sufficiently large, the lower hedging costs in the monopolistic case can outweigh the competition effect in the duopolistic case, and investors are better off with a monopolistic market maker. The following proposition formalizes these results.

Proposition 4. *Suppose that:*

$$\left(1 + \gamma^2 \, \sigma_\epsilon^2 \, \sigma_x^2\right) \sigma_\epsilon^4 < 1. \tag{32}$$

Then, there exists a correlation coefficient $\rho^* \in (0, 1)$ *such that, for all* $|\rho| \in (0, \rho^*)$*, the investors' ex ante expected utility (before signals and endowments are observed) is higher under the monopolistic market-making scenario. For all* $|\rho| \in (\rho^*, 1)$*, investors strictly prefer the duopolistic market-making scenario.*

If the above condition is not satisfied, investors are better off under the duopolistic market-making scenario for all $|\rho| \in (0, 1)$*.*

Intuitively, the condition in (32) requires the information that market makers can infer from the observed order flow to be sufficiently precise. If investors are not very risk averse (low γ) and/or observe precise signals about asset payoffs (low σ_ϵ), they trade aggressively on their private information, thus making the order flow highly informative about the payoff vector v. Similarly, a low variance of the investors' endowment shocks, σ_x^2, means that the investors' demand is more closely correlated with their signals s rather than their payoff-irrelevant endowment $\bar{x}$. In these cases, the monopolist's informational advantage from observing the demand for two correlated assets leads to a sufficient reduction in the investors' hedging costs that outweighs their benefit from having market makers compete for order flow.

The critical correlation coefficient ρ^*, up to which investors prefer the monopolistic scenario, is characterized by a polynomial of degree 10. Thus, no closed-form solution is available and we have to numerically analyze how ρ^* varies with the model primitives γ, σ_ϵ^2, and σ_x^2. Figures 1 and 2 show that the critical correlation coefficient ρ^* decreases with the investors' risk aversion coefficient γ and their signal noise σ_ϵ^2.[31] This is not surprising. As investors become more risk averse and/or receive less precise signals, they trade less aggressively on their private information. This reduces the informational advantage of the monopolistic market maker, making the more competitive prices in the duopolistic scenario more appealing to investors.

An increase in the variance of stock endowments has a similar effect. A larger expected endowment shock (in either direction) means that a larger part of the investors' trades is motivated by portfolio rebalancing needs rather than private information. This again makes the observed order flow less informative about the stock's payoff, thereby reducing the monopolist's informational advantage over the duopolist. Thus, the duopolistic scenario becomes more attractive to investors even at lower levels of asset correlation within an industry. In other words, ρ^* is a decreasing function of σ_x^2 (see Figure 3).

4.3 Liquidity and trading volume

The above discussion shows that the allocation of securities between market makers can have a significant effect on the investors' trading costs. However, investors have no direct influence on the allocation decision. Rather, this decision is made by the exchange, typically in collaboration with the listed firm.[32] While an exchange may use the allocation process to pursue a variety of objectives, maintaining a high level of liquidity in share trading seems to be an important one. For example, Corwin (2004) reports that the NYSE is more likely to allocate new listings to specialist firms with more frequent price improvements and greater

[31]The comparative static results in Figures 1 to 3 are based on the parameter values $\gamma = 1$, $\sigma_\epsilon^2 = 1/2$, and $\sigma_x^2 = 1$, which satisfy the condition in (32). We want to emphasize, however, that the results are robust to changes in parameter values. They continue to hold for a range of values around the chosen value for each parameter.

[32]On the NYSE, for example, listing firms have had the option to choose their preferred specialist from a subset of specialist units selected by the exchange's Allocation Committee since 1997. Prior to that, the allocation decision was made solely by the exchange.

quoted depth. A more liquid market also benefits the listed firm. Several empirical studies show that liquidity is priced in the cross-section of expected stock returns: firms with more liquid shares face a lower cost of equity capital.[33]

Liquidity is generally defined as the ability to trade large quantities quickly with little price impact. In our model, the price impact of an order is determined by the price coefficients δ^M and δ^D. In particular, an increase in uninformed demand by dx_n shares for a stock results in a price change of:[34]

$$dP_n = \gamma \Sigma_{v|s,11} \left(\delta_{11} + \rho_{v|s} \delta_{12} \right) dx_n, \tag{33}$$

where δ_{ij} denotes the (i, j)th entry of the matrix δ^M (in the monopoly case) or δ^D (in the duopoly case). This expression measures by how much the order flow in an asset moves its own price. Of course, changes in the demand for an asset may also lead to price changes in other assets if their payoffs are correlated. While these price changes affect the informational efficiency of equilibrium prices, they are not manifestations of the liquidity of an asset.

The following proposition shows that the level of liquidity, as measured by (the inverse of) the magnitude of the price impact in (33), is closely linked to investors' welfare.

Proposition 5. *If the condition in Proposition 4 is satisfied and $|\rho| \in (0, \rho^*)$, then liquidity is higher under the monopolistic market-making scenario. For values of $|\rho|$ close to one, liquidity is higher under the duopolistic market-making scenario.*

Proposition 5 states that specialists provide more liquidity under the monopolistic scenario whenever investors prefer this arrangement. However, the converse is not true. There exist parameter values for which investors are better off under the duopolistic scenario even though assets are less liquid in this case. The reason is that the liquidity measure specified above does not take into account how the order flow in an asset affects prices of other assets. Thus,

[33] See, e.g., Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Amihud (2002), and Chordia, Huh, and Subrahmanyam (2009).

[34] Since the price coefficients are identical for both assets, this expression reflects the price change of either asset caused by an infinitesimal increase in its own order flow.

prices quoted by a monopolistic specialist may be less sensitive to the assets' own order flow, but their correlation with the order flow in other assets can make investors worse off.

The intuition for the above result is similar to the one given for Proposition 4. Compared to the duopolistic case, a monopolistic market maker can learn more about the value of an asset by observing the order flow in other assets. As such, she changes her beliefs to a smaller extent in response to an asset's order flow. If markets were perfectly competitive, this would mean that the price impact of an order is comparatively smaller in the monopolistic scenario. However, since markets are imperfectly competitive, a monopolistic market maker also exploits her market power by making her prices more sensitive to the observed order flow, thereby reducing liquidity. Which effect dominates depends on the correlation of asset payoffs. If the monopolist's informational advantage is sufficiently large (low γ, σ_ϵ, and/or σ_x) and the competition between specialists' is relatively weak (low $|\rho|$), the information effect dominates the competition effect and liquidity is higher under the monopolistic market-making scenario.

In many cases, exchanges compensate their specialists based on the number of shares that they trade.[35] This suggests that trading volume may be an alternative measure of liquidity that influences the security allocation decision of an exchange. Given the market makers' trading strategies, trading volume in our model can be calculated from changes in the holdings of either investors or dealers. For simplicity, we consider the trades of investors:

$$V_1 = |x_1(P)| = \kappa \left| \left(1 - \delta_{11} + \rho_{v|s}\delta_{12}\right)\theta_1 - \left(\rho_{v|s}(1-\delta_{22}) + \delta_{12}\right)\theta_2 \right|, \tag{34}$$

where δ_{ij} again denotes the (i,j)th entry of the matrix δ^M (in the monopoly case) or δ^D (in the duopoly case) and κ is a positive constant. Since θ_1 and θ_2 are jointly normally distributed with zero means, the expected trading volume is given by:

$$\mathbb{E}\left[V_1\right] = \kappa \sqrt{\frac{2}{\pi} \mathrm{Var}\left[\left(1 - \delta_{11} + \rho_{v|s}\delta_{12}\right)\theta_1 - \left(\rho_{v|s}(1-\delta_{22}) + \delta_{12}\right)\theta_2\right]}. \tag{35}$$

[35]The NYSE, for example, pays 30 cents per 100 traded shares to designated market makers.

An analogous expression can be derived for asset 2. In fact, since θ_1 and θ_2 are identically distributed and the price coefficient matrices are symmetric, the expected trading volume of asset 2 is the same as that of asset 1.

Intuitively, one might expect that the more competitive prices under the duopolistic market-making scenario encourage investors to trade larger quantities. This is, however, not the case as the following proposition shows.

Proposition 6. *The expected trading volume is higher under the monopolistic market-making scenario than under the duopolistic scenario for all* $|\rho| \in (0, 1)$.

This result is closely related to our discussion of how prices are affected by the market makers' information. As argued in Section 4.2, prices quoted by better informed monopolistic specialists are less (negatively) correlated with the investors' stock endowments. This allows investors to liquidate a larger portion of their endowment without much price impact. Of course, this reduction in hedging costs has to be weighed against higher price mark-ups in the monopoly case due to the specialist's increased market power. In equilibrium, the former effect dominates, leading to a higher trading volume under the monopolistic market-making scenario.

5 Conclusion

This paper develops a noisy rational expectations model of a specialist market that enables us to address the question of how securities with correlated payoffs should be allocated to specialists. We compare the welfare of the different parties under two market-making scenarios. In the first scenario, each market maker is in charge of two stocks in the same industry. Since security payoffs are assumed to be independent across industries, there is no competition for order flow in this situation. Thus, each market maker acts as a monopolist when setting her prices. We refer to this case as the monopolistic market-making scenario. In the second scenario, which we call the duopolistic market-making scenario, each market maker is assigned one stock from each of the two industries. We show that, for both market-making scenarios, there exists a unique linear PBE and we derive closed-form expressions for the equilibrium prices.

Our analysis demonstrates that specialists always prefer portfolios of securities with highly correlated payoffs. Their expected profit in the monopolistic scenario exceeds that in the duopolistic scenario for any level of within-industry correlation. This is due to their increased market power as well as their privileged access to information when they have control over several securities in an industry.

For investors, the trade-off between these two market-making scenarios is more complicated. On one hand, they benefit from the specialists' ability to better distinguish between informational and non-informational trades in the monopolistic scenario, which reduces their hedging costs. On the other hand, they suffer from a lack of competition in this case. We show that, for high levels of within-industry correlation, the adverse price effect due to the specialists' increased market power dominates the reduction in hedging costs and investors prefer the duopolistic market-making scenario. For low levels of correlation, however, investors may be better off with a monopolist specialist. This is typically the case when the investors' order flow is highly informative about the value of the traded securities.

Finally, we discuss how these two different security allocations to specialists affect various measures of market liquidity. We show that the average trading volume is higher when securities in the same industry are allocated to the same specialist. The price impact of an order is typically higher under the monopolistic market-making scenario for high values of within-industry payoff correlation, and higher under the duopolistic scenario for low values of payoff correlation. This is consistent with the empirical evidence that stock reassignments from less concentrated to more concentrated specialist portfolios are generally associated with an improvement in market liquidity on the NYSE (Anand, Chakravarty, and Chuwonganant, 2009).

Appendix

Proof of Proposition 2. Since investors are price takers, their optimal demand functions are independent of the market makers' strategies and given by equation (12). Thus, we are left to show that the strategy profile specified by equation (28) is the unique linear equilibrium of the Cournot duopoly game and that the prices defined by equation (29) are implied by this strategy profile and the investors' inverse demand function (i.e., $P^D = x^{-1}(\hat{x})$).

From our analysis in Section 3.2.2, we know that the market makers' best response to the expected strategy of their opponent is given by equations (26) and (27), respectively. Now suppose that these strategies are linear in the market makers' signal, i.e.:

$$\hat{x}_m = \phi_m \, \tau_m, \qquad m \in \{1, 2\}. \tag{36}$$

Since τ_1 and τ_2 are jointly normally distributed, the conditional expectation of $\hat{x}_2$ given τ_1 is then equal to:

$$\mathbb{E}\left[\hat{x}_2 | \tau_1\right] = \phi_2 \, \omega_1^{\mathrm{T}} \, \Sigma_\theta \, \omega_2 \left(\omega_1^{\mathrm{T}} \, \Sigma_\theta \, \omega_1\right)^{-1} \, \tau_1, \tag{37}$$

where $\omega_1 = (1, -\rho_{v|s})^{\mathrm{T}}$ and $\omega_2 = (-\rho_{v|s}, 1)^{\mathrm{T}}$ as before. Substituting this expression into the first order condition in (26), we have:

$$\hat{x}_1 = \frac{1}{2 \, \omega_1^{\mathrm{T}} \, \Sigma_\theta \, \omega_1} \left(\frac{\omega_1^{\mathrm{T}} (\Sigma_\theta - \Sigma_\mu) \, e_1}{\gamma \, \sigma_{v|s}^2} - \rho_{v|s} \, \phi_2 \, \omega_1^{\mathrm{T}} \, \Sigma_\theta \, \omega_2 \right) \tau_1. \tag{38}$$

Similarly, for market maker 2 we obtain:

$$\hat{x}_2 = \frac{1}{2 \, \omega_2^{\mathrm{T}} \, \Sigma_\theta \, \omega_2} \left(\frac{\omega_2^{\mathrm{T}} (\Sigma_\theta - \Sigma_\mu) \, e_2}{\gamma \, \sigma_{v|s}^2} - \rho_{v|s} \, \phi_1 \, \omega_1^{\mathrm{T}} \, \Sigma_\theta \, \omega_2 \right) \tau_2. \tag{39}$$

Equating the coefficient of τ_1 (respectively, τ_2) in equation (38) (respectively, equation (39)) to ϕ_1 (respectively, ϕ_2) and solving the resulting two linear equations for ϕ_1 and ϕ_2 yields:

$$\phi_m = \frac{\omega_m^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\, e_m}{\gamma\, \sigma_{v|s}^2 \left(2\, \omega_m^{\mathrm{T}}\, \Sigma_\theta\, \omega_m + \rho_{v|s}\, \omega_1^{\mathrm{T}}\, \Sigma_\theta\, \omega_2\right)}, \qquad m \in \{1, 2\}, \tag{40}$$

where we have used the fact that $\omega_1^{\mathrm{T}}\, \Sigma_\theta\, \omega_1 = \omega_2^{\mathrm{T}}\, \Sigma_\theta\, \omega_2$ and that $\omega_1^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\, e_1 = \omega_2^{\mathrm{T}}(\Sigma_\theta - \Sigma_\mu)\, e_2$. This proves that the strategy profile $\hat{x} = (\phi_1\, \tau_1, \phi_2\, \tau_2)$ is the unique linear equilibrium.

The equilibrium prices P^D follow immediately from the market makers' equilibrium strategies and the investors' inverse demand function:

$$\begin{aligned} P_1^D &= x_1^{-1}(\hat{x}) & (41)\\ &= \theta_1 - \gamma\, \sigma_{v|s}^2 \left(\phi_1\, \tau_1 + \rho_{v|s}\, \phi_2\, \tau_2\right) & (42)\\ &= \left(1 - \gamma\, \sigma_{v|s}^2 \left(1 - \rho_{v|s}^2\right) \phi_1\right) \theta_1. & (43) \end{aligned}$$

Analogous calculations show that $P_2^D = \left(1 - \gamma\, \sigma_{v|s}^2 \left(1 - \rho_{v|s}^2\right) \phi_2\right) \theta_2$. ■

Proof of Proposition 3. We prove this proposition in two steps. First, we show that, for any $\rho \neq 0$, the specialist's ex ante expected profit from making a market for two stocks of a given industry in the monopolistic scenario strictly exceeds her expected profit in the fictitious scenario in which she follows the duopolistic trading strategies for the same two stocks. Next, we argue that her expected profit in this fictitious scenario is the same as that in the duopolistic scenario.

To prove the first result, note that the market maker's information set in the duopolistic scenario is a subset of her information set in the monopolistic scenario: the realization of $\{\tau_1, \tau_2\}$ can be inferred from the realization of $\{\theta_1, \theta_2\}$, but not vice versa. Thus, a monopolistic market maker can choose to mimic the duopolistic strategy and quote a price of P^D if it is optimal for her to do so. However, as the expressions for δ^M and δ^D in equations (18) and (30) show, this is not the case for any $\rho \neq 0$. This means that the market maker's expected profit conditional on any realization of $\{\theta_1, \theta_2\}$ and, hence, also her ex ante expected profit is strictly larger in the monopolistic scenario than in the fictitious scenario in which she follows the duopolistic trading strategy for the same two stocks. Clearly, if $\rho = 0$, the investors' demand for the two stocks is independent and the two market-making scenarios are identical.

The second result follows immediately from the fact that all securities are ex ante identical, that the within-industry correlation is the same for both industries, and that the investors' demand functions x_a and x_b are independent. This proves that the specialist's ex ante expected profit in the monopolistic market-making scenario strictly exceeds that in the duopolistic scenario if $\rho \neq 0$. ■

Lemma 1. *Suppose that $P = \delta\,\theta$. Then the investors' ex ante expected utility is equal to:*

$$EU_I = \frac{1}{\gamma}\,\mathrm{tr}\left[\Sigma_\mu\,\Sigma_{v|s}^{-1} - \Sigma_\theta\,(2I_2 - \delta)\,\Sigma_{v|s}^{-1}\,\delta\right], \tag{44}$$

where $\mathrm{tr}[A]$ *denotes the trace of matrix A.*

Proof of Lemma 1. Since the two industries a and b are independent and ex ante identical, the investors' ex ante expected utility from trading shares of firms in both industries is equal to two times the expected utility from trading shares of firms in only one industry. Thus, we have:

$$EU_I = \mathbb{E}\left[2\,(x(P)+\bar{x})^{\mathrm{T}}\,(\mu - P) + 2\,\bar{x}^{\mathrm{T}}P - \gamma\,(x(P)+\bar{x})^{\mathrm{T}}\,\Sigma_{v|s}\,(x(P)+\bar{x})\right]. \tag{45}$$

Substituting the investors' demand function into this expression and replacing the equilibrium prices P by $\delta\theta$, we get:

$$EU_I = \frac{1}{\gamma}\,\mathbb{E}\left[(\mu - \delta\theta)^{\mathrm{T}}\,\Sigma_{v|s}^{-1}\,(\mu - \delta\theta) + 2\,\gamma\,\bar{x}^{\mathrm{T}}\delta\theta\right]. \tag{46}$$

Using the fact that $\theta = \mu - \gamma\,\Sigma_{v|s}\,\bar{x}$, we can further simplify this expression to:

$$EU_I = \frac{1}{\gamma}\,\mathbb{E}\left[\mu^{\mathrm{T}}\Sigma_{v|s}^{-1}\,\mu - \theta^{\mathrm{T}}\,(2I_2 - \delta)\,\Sigma_{v|s}^{-1}\,\delta\theta\right] \tag{47}$$

$$= \frac{1}{\gamma}\,\mathrm{tr}\left[\Sigma_\mu\,\Sigma_{v|s}^{-1} - \Sigma_\theta\,(2I_2 - \delta)\,\Sigma_{v|s}^{-1}\,\delta\right], \tag{48}$$

where tr$[A]$ denotes the trace of matrix A. ■

Proof of Proposition 4. In this proof, some complex expressions are omitted for brevity. The details are available from the author upon request.

Let ΔEU_I denote the difference between the investors' ex ante expected utility in the monopolistic and the duopolistic market-making scenario, i.e., $\Delta EU_I = EU_I^M - EU_I^D$. Using the result in Lemma 1, this difference can be expressed as the quotient of two polynomials of degree 8 in ρ:

$$\Delta EU_I = \frac{c_2\,\rho^2 + c_4\,\rho^4 + c_6\,\rho^6 + c_8\,\rho^8}{d_0 + d_2\,\rho^2 + d_4\,\rho^4 + d_6\,\rho^6 + d_8\,\rho^8}, \tag{49}$$

where the coefficients $c_2, \ldots, c_8$ and $d_0, \ldots, d_8$ are functions of γ, σ_ϵ, and σ_x. It can be shown that the denominator is strictly positive for all $\rho \in [-1, 1]$. Thus, ΔEU_I is a continuous function of ρ over the interval $[-1, 1]$. Furthermore, since all coefficients of uneven powers of the two polynomials are equal to zero, ΔEU_I is symmetric around $\rho = 0$.

Next, we show that the numerator polynomial has at most one root in the interval $(0, 1)$. In order to see this, note that the nonzero roots of this polynomial are given by $\rho = \pm\sqrt{z}$, where z is a root of the cubic function:

$$f(z) = c_2 + c_4\,z + c_6\,z^2 + c_8\,z^3. \tag{50}$$

It is straightforward to show that the discriminant of this cubic function, which is given by:

$$18\,c_2\,c_4\,c_6\,c_8 - 4\,c_2\,c_6^3 + c_4^2\,c_6^2 - 4\,c_4^3\,c_8 - 27\,c_2^2\,c_8^2, \tag{51}$$

is strictly negative, which implies that it has a unique real root. This, of course, means that the numerator polynomial in (49) has at most one positive root.

For a root in the interval $(0, 1)$ to exist, it suffices to show that $f(0) > 0$ and that $f(1) < 0$. It can easily be verified that the latter is true for all parameter values. Since the coefficient c_2 is proportional to $1 - \left(1 + \gamma^2\, \sigma_\epsilon^2\, \sigma_x^2\right) \sigma_\epsilon^4$, the former is true if and only if:

$$\left(1 + \gamma^2\, \sigma_\epsilon^2\, \sigma_x^2\right) \sigma_\epsilon^4 < 1. \tag{52}$$

This proves that, if the above inequality holds, there exists a correlation coefficient $\rho^* \in (0, 1)$ such that $\Delta EU_I > 0$ for all $0 < |\rho| < \rho^*$ and $\Delta EU_I < 0$ for all $\rho^* < |\rho| < 1$. If this inequality does not hold, $\Delta EU_I < 0$ for all $0 < |\rho| < 1$. ■

Proof of Proposition 5. As our discussion in Section 4.3 shows, the price impact of an uninformed order in the monopolistic scenario exceeds that in the duopolistic scenario if and only if:

$$\Delta PI \equiv \delta_{11}^M + \rho_{v|s} \delta_{12}^M - \delta_{11}^D > 0, \tag{53}$$

where we have used the fact that $\delta_{12}^D = 0$. Substituting the equilibrium price coefficients from equations (18) and (30) into this expression, it is straightforward to show that ΔPI is positive when $|\rho| = 1$. This proves that, for large enough values of $|\rho|$, liquidity is higher (i.e., the price impact is lower) in the duopolistic market-making scenario.

For low values of $|\rho|$, ΔPI can be negative if the following condition holds:

$$\left(1 + \gamma^2\, \sigma_\epsilon^2\, \sigma_x^2\right) \sigma_\epsilon^4 < 2 + \sigma_\epsilon^2. \tag{54}$$

In particular, it can be shown that, if the above inequality holds, there exists a $\hat{\rho} > \rho^*$ such that $\Delta PI < 0$ for all $|\rho| \in (0, \hat{\rho})$.[36] Since the condition in (54) is weaker than the condition

[36]Since the expressions involved are rather complex, we do not present a formal proof of this claim here. A detailed proof can be obtained from the author upon request.

in Proposition 4 (in the sense that the former is satisfied whenever the later is satisfied), this implies that liquidity is higher (i.e., the price impact is lower) in the monopolistic market-making scenario whenever investors prefer this arrangement. ■

Proof of Proposition 6. Equation (35) shows that the expected trading volume in a stock is given by:

$$\mathbb{E}\left[V_n\right] = \kappa\sqrt{\frac{2}{\pi}\,\xi^{\mathrm{T}}\Sigma_\theta\,\xi}, \tag{55}$$

where κ is a positive constant and:

$$\xi = \begin{pmatrix} 1-\delta_{11}+\rho_{v|s}\delta_{12} \\ -\rho_{v|s}(1-\delta_{22})-\delta_{12} \end{pmatrix}. \tag{56}$$

Substituting the equilibrium price coefficients δ^M and δ^D into the above expression reveals (after some tedious but straightforward algebra) that the expected trading volume in the monopolistic scenario exceeds that in the duopolistic scenario for all $|\rho| \in (0,1)$. ■

References

Admati, A. R., 1985, "A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets," *Econometrica*, 53, 629–657.

Amihud, Y., 2002, "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects," *Journal of Financial Markets*, 5, 31–56.

Amihud, Y., and H. Mendelson, 1986, "Asset Pricing and the Bid-Ask Spread," *Journal of Financial Economics*, 17, 223–249.

Anand, A., S. Chakravarty, and C. Chuwonganant, 2009, "Cleaning House: Stock Reassignments on the NYSE," *Journal of Financial Markets*, 12, 727–753.

Anand, A., and A. Subrahmanyam, 2008, "Information and the Intermediary: Are Market Intermediaries Informed Traders in Electronic Markets?," *Journal of Financial and Quantitative Analysis*, 43, 1–28.

Anderson, T. W., 1984, *An Introduction to Multivariate Statistical Analysis*, John Wiley & Sons, New York, 2nd edn.

Baruch, S., G. A. Karolyi, and M. L. Lemmon, 2007, "Multimarket Trading and Liquidity: Theory and Evidence," *Journal of Finance*, 62, 2169–2200.

Baruch, S., and G. Saar, 2009, "Asset Returns and the Listing Choice of Firms," *Review of Financial Studies*, 22, 2239–2274.

Benveniste, L. M., A. J. Marcus, and W. J. Wilhelm, 1992, "What's Special About the Specialist?," *Journal of Financial Economics*, 32, 61–86.

Bernhardt, D., and B. Taub, 2008, "Cross-Asset Speculation in Stock Markets," *Journal of Finance*, 63, 2385–2427.

Biais, B., D. Martimort, and J.-C. Rochet, 2000, "Competing Mechanisms in a Common Value Environment," *Econometrica*, 68, 799–837.

Brennan, M. J., and A. Subrahmanyam, 1996, "Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns," *Journal of Financial Economics*, 41, 441–464.

Caballé, J., and M. Krishnan, 1994, "Imperfect Competition in a Multi-Security Market with Risk Neutrality," *Econometrica*, 62, 695–704.

Charitou, A., and M. Panayides, 2008, "Market Making in International Capital Markets: Challenges and Benefits of its Implementation in Emerging Markets," Working Paper, University of Utah.

Chordia, T., S.-W. Huh, and A. Subrahmanyam, 2009, "Theory-Based Illiquidity and Asset Pricing," *Review of Financial Studies*, 22, 3629–3668.

Chowdhry, B., and V. Nanda, 1991, "Multimarket Trading and Market Liquidity," *Review of Financial Studies*, 4, 483–511.

Corwin, S. A., 2004, "Specialist Performance and New Listing Allocations on the NYSE: An Empirical Analysis," *Journal of Financial Markets*, 7, 27–51.

Gehrig, T., and M. Jackson, 1998, "Bid-Ask Spreads with Indirect Competition among Specialists," *Journal of Financial Markets*, 1, 89–119.

Glosten, L. R., 1989, "Insider Trading, Liquidity, and the Role of the Monopolist Specialist," *Journal of Business*, 62, 211–235.

Hagerty, K., 1991, "Equilibrium Bid-Ask Spreads in Markets with Multiple Assets," *Review of Economic Studies*, 58, 237–257.

Harris, L. E., and V. Panchapagesan, 2005, "The Information Content of the Limit Order Book: Evidence from NYSE Specialist Trading Decisions," *Journal of Financial Markets*, 8, 25–67.

Hasbrouck, J., and G. Sofianos, 1993, "The Trades of Market Makers: An Empirical Analysis of NYSE Specialists," *Journal of Finance*, 48, 1565–1593.

Hirshleifer, J., 1971, "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review*, 61, 561–574.

Kremer, I., and V. Polkovnichenko, 2000, "Competition in Financial Dealership Markets," Working Paper, University of Minnesota.

Kreps, D. M., and J. A. Scheinkman, 1983, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics*, 14, 326–337.

Kyle, A. S., 1985, "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315–1336.

New York Stock Exchange, Inc., 1998, *Information Memo 98-06: Allocation Policy and Procedures*, January 30, 1998.

New York Stock Exchange, Inc., 2001, *Information Memo 01-37: Allocation Policy*, October 29, 2001.

Ready, M. J., 1999, "The Specialist's Discretion: Stopped Orders and Price Improvement," *Review of Financial Studies*, 12, 1075–1112.

Seligman, J., 2003, *The Transformation of Wall Street: A History of the Securities and Exchange Commission and Modern Corporate Finance*, Aspen Publishers, New York, 3rd edn.

Skjeltorp, J. A., and B. A. Ødegaard, 2012, "Why Do Listed Firms Pay for Market Making in Their Own Stock?," Working Paper, Norges Bank and University of Stavanger.

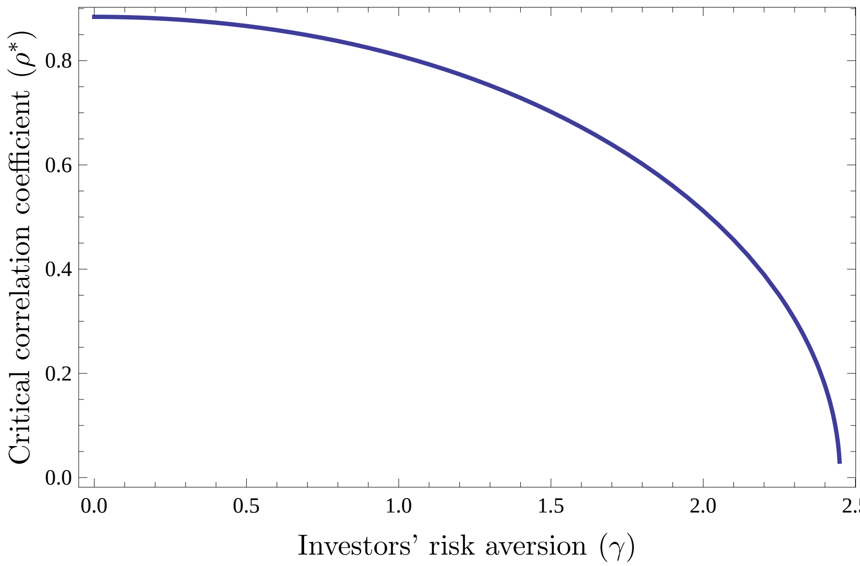

Figure 1: The graph presents the critical correlation coefficient ρ^* as a function of the investors' risk aversion coefficient γ. The parameter values used in the graph are $\sigma_\epsilon^2 = 1/2$ and $\sigma_x^2 = 1$.

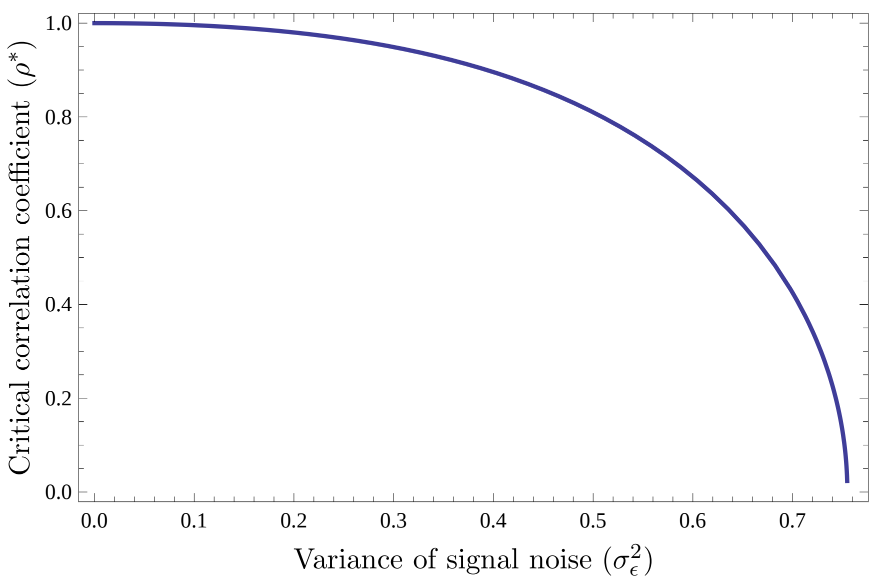

Figure 2: The graph presents the critical correlation coefficient ρ^* as a function of the variance of the investors' signal noise σ_ϵ^2. The parameter values used in the graph are $\gamma = 1$ and $\sigma_x^2 = 1$.

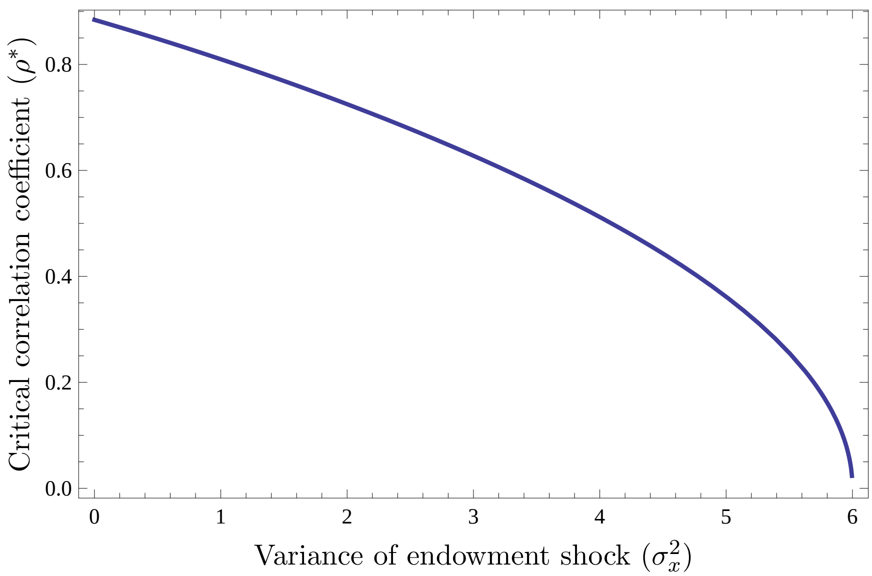

Figure 3: The graph presents the critical correlation coefficient ρ^* as a function of the variance of the investors' endowment shock σ_x^2. The parameter values used in the graph are $\gamma = 1$ and $\sigma_\epsilon^2 = 1/2$.